RELATIVITY

1. Special Relativity
2. Time Dilation
3. Doppler Effect
4. Length Contraction
5. Twin Paradox
6. Electricity and Magnetism
7. Relativistic Momentum
8. Mass and Energy
9. Energy and Momentum
10. General Relativity

MASS AND ENERGY

The most famous relationship Einstein obtained from the postulates of special relativity was that links mass with energy.

From where does this relation come about?

MASS AND ENERGY

What is work?

Work W done on an object by a constant force of magnitude F that acts through a distance s, where F is in the same direction as s is:

$W = F\,s$

What is kinetic energy?

If no other forces act on the object and the object starts from rest, then all work done on it becomes kinetic energy KE:

$\mathrm{KE} = F\,s$

Since F need not be constant then:

$$\mathrm{KE} = \int_0^s F\,ds$$

MASS AND ENERGY

What is KE in non-relativistic physics?

$$\mathrm{KE} = \frac{1}{2} m\upsilon^2$$

What is KE relativistically?

We start from the second law of motion:

$$\mathrm{KE} = \int_0^s \frac{d(\gamma\, m\, \upsilon)}{dt}\, ds = \int_0^{m\upsilon} \upsilon\, d(\gamma\, m\, \upsilon) = \int_0^{\upsilon} \upsilon\, d\Big(\frac{m\,\upsilon}{\sqrt{1-\upsilon^2/c^2}}\Big)$$

Integrating by parts:

$$\int x\,dy = xy - \int y\,dx$$

$$\mathrm{KE} = \frac{m\,\upsilon^2}{\sqrt{1-\upsilon^2/c^2}} - m\int_0^{\upsilon} \frac{\upsilon\, d\upsilon}{\sqrt{1-\upsilon^2/c^2}}$$

$$= \frac{m\,\upsilon^2}{\sqrt{1-\upsilon^2/c^2}} + \Big[mc^2\sqrt{1-\upsilon^2/c^2}\Big]_0^{\upsilon}$$

$$= \frac{m\,c^2}{\sqrt{1-\upsilon^2/c^2}} - m\,c^2$$

$$= \gamma\, m\, c^2 - mc^2$$

$$\mathrm{KE} = (\gamma - 1) m\, c^2$$

MASS AND ENERGY

What is the total energy?

$$\mathrm{E} = \gamma m c^2 = m c^2 + KE$$

Even when the particle at rest it still possesses energy.

What is the rest energy?

$$\mathrm{E}_0 = m c^2$$

$$\mathrm{E} = E_0 + KE$$

$$\mathrm{E} = \gamma m c^2 = \frac{m c^2}{\sqrt{1 - \upsilon^2 / c^2}}$$

Since mass and energy are not independent, their conservation principle is a single one –

the principle of conservation of mass energy

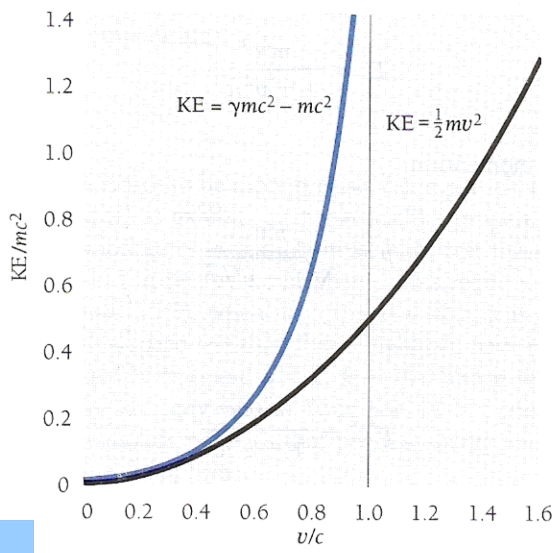
MASS AND ENERGY

Will the relativistic formula for KE reduce to the classical formula?

$$\mathrm{KE} = \gamma m c^2 - mc^2 = \frac{m c^2}{\sqrt{1-\upsilon^2/c^2}} - m c^2$$

Since $\upsilon^2/c^2 << 1$ we can use the binomial approximation:

$$(1+x)^n \approx 1 + nx$$



$$\frac{1}{\sqrt{1-\upsilon^2/c^2}} \approx 1 + \frac{1}{2}\frac{\upsilon^2}{c^2} \quad , \quad \upsilon << c$$

$$\mathrm{KE} \approx \left(1 + \frac{1}{2}\frac{\upsilon^2}{c^2}\right) mc^2 - mc^2 \approx \frac{1}{2} m\upsilon^2 \quad , \quad \upsilon << c$$

MASS AND ENERGY

Remember…

Mass and energy are connected through $E_o = m\,c^2$ …

MASS AND ENERGY

Example 1.6:

A stationary body explodes into two fragments each of mass 1.0 kg that move apart at speeds of 0.6c relative to the original body. Find the mass of the original body.

MASS AND ENERGY

Example 1.7:

Solar energy reaches the earth at the rate of about 1.4 kW per square meter of surface perpendicular to the direction of the sun. By how much does the mass of the sun decrease per second owing to this energy loss? The mean radius of the earth's orbit is 1.5×10^{11} m