Simple Pendulum - Worksheet

Watch the videos and answer the following questions:

1. Describe the motions you saw in the videos?

2. What are the forces that cause the motion of the child and the clock?

3. The equipment in front of you is called a simple pendulum and it exhibits a simple harmonic motion. The period of the pendulum is the time required to complete one complete cycle. Measure the period of the simple pendulum and record your reading.

4. Try to discover the factors that affect the motion of the simple pendulum? Write you observation.

5. At a specific length, how is the period T is affected by a change in mass?

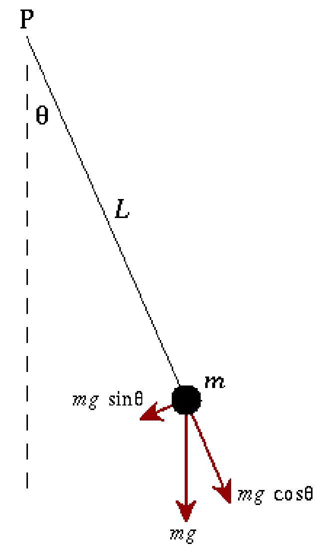
6. Change the length of the pendulum and record the period at the different lengths. Tabulate your results

7. It is suggested that the relation between the length of the pendulum and its period is given by the relation $T^2 = 4\pi^2 L/g$, where g is the acceleration due to gravity. Using the data obtained in <u>step 6</u> check the validity of this relation.

8. What should you do to the length of the string of a simple pendulum to double its period?

9. You are captured by Martians, taken into their ship, and put to sleep. You wake some time later and find yourself locked in a small room with no windows. All the Martians have left you with is your digital watch, your school ring, and your long silver-chain necklace. Explain how you can determine whether you are still on earth or have been transported to Mars?

Simple Pendulum



A simple pendulum consists of a mass m hanging from a string of length L and fixed at a pivot point P. When displaced to an initial angle and released, the pendulum will swing back and forth with periodic motion. By applying Newton's second law for rotational systems, the equation of motion for the pendulum may be obtained

$$\tau = I\,\alpha \;\rightarrow\; -mg\sin\theta\;L = mL^2\,\frac{d^2\theta}{dt^2}$$

and rearranged as

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$

If the amplitude of angular displacement is small enough that the small angle approximation ($\sin\theta \approx \theta$) holds true, then the equation of motion reduces to the equation of simple harmonic motion

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

The simple harmonic solution is

$$\theta(t) = \theta_0\cos(\omega t + \varphi)$$

with $\omega = \sqrt{g/L}$ being the natural frequency of the motion.

Small Angle Approximation and Simple Harmonic Motion

With the assumption of small angles, the frequency and period of the pendulum are independent of the initial angular displacement amplitude. All simple pendulums should have the same period regardless of their initial angle (and regardless of their masses). The small angle approximation is valid for initial angular displacements of about 20° or less.

In the case of small angles, the period for a simple pendulum does not depend on the mass or the initial angular displacement, but depends only on the length L of the string and the value of the gravitational field strength g, according to

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When the angular displacement amplitude of the pendulum is large enough that the small angle approximation no longer holds, the equation of motion must remain in its nonlinear form

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$

This differential equation does not have a closed form solution, and must be solved numerically using a computer.