

# MATHEMATICA ABC

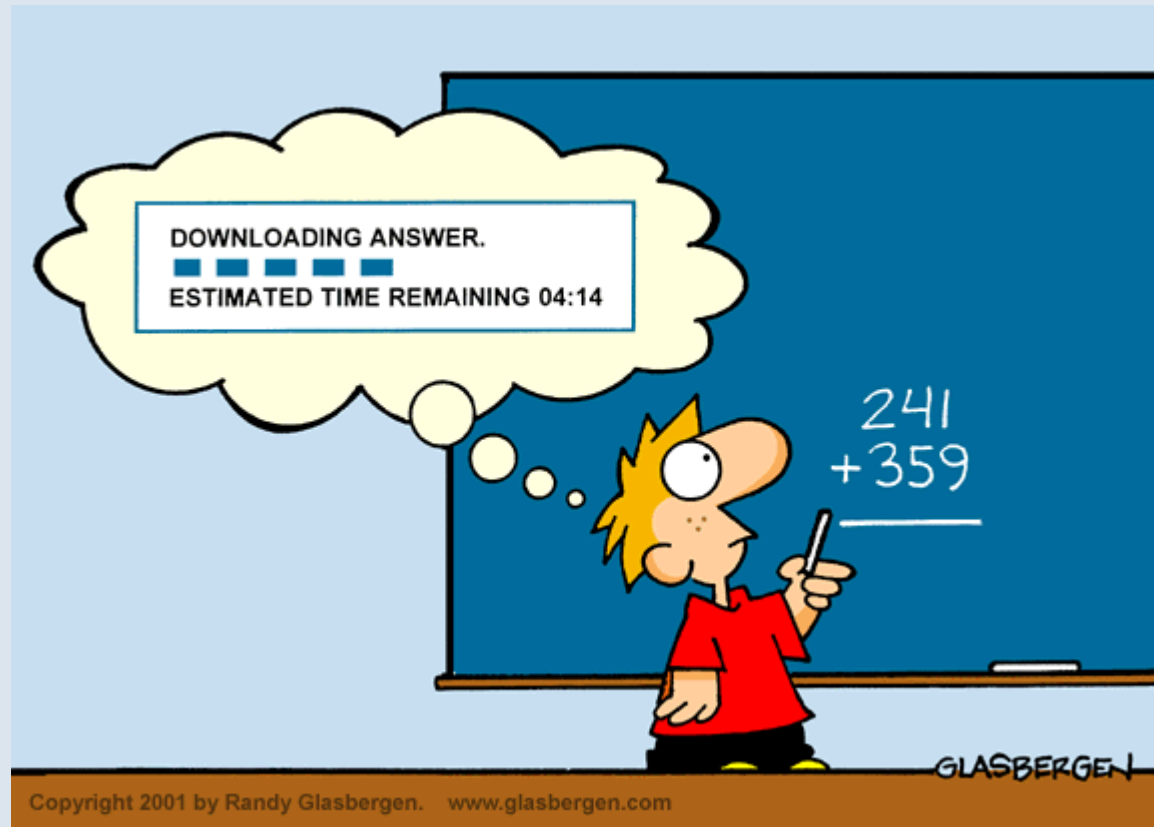
TRAINIER: DR REEM ALTUWIRQI

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# Timetable and Syllabus...

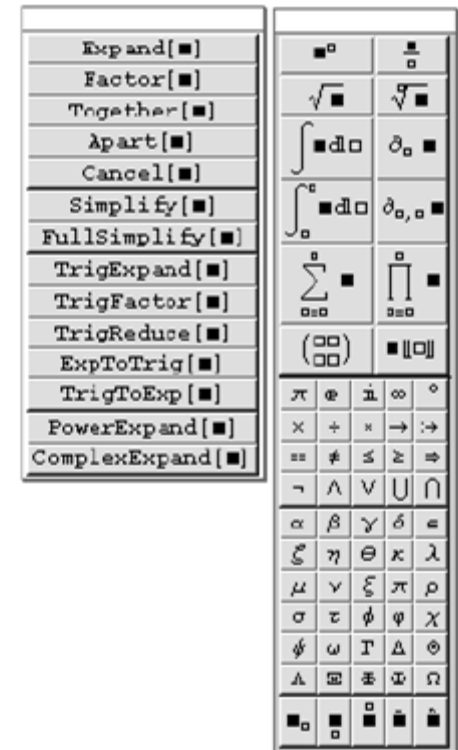
Session 1	Session 2	Session 3	Session 4	Session 5
Getting Started	User defined functions and plotting	Calculus	Vector analysis	Examples and Closing
<ul style="list-style-type: none"><li>- Syntax</li><li>- Packages</li><li>- Help</li><li>- Numerical calculations</li><li>- Built-in constants and functions</li><li>- Word processing</li></ul>	<ul style="list-style-type: none"><li>- Variables, equations and functions</li><li>- Plotting functions</li><li>- Solving equations and inequalities</li></ul>	<ul style="list-style-type: none"><li>- Limits</li><li>- Derivatives</li><li>- Integration</li></ul>	<ul style="list-style-type: none"><li>- Series</li><li>- Sum</li><li>- Matrices</li></ul>	<ul style="list-style-type: none"><li>- Some examples in physics</li><li>- Course project</li><li>- Closing</li></ul>
Worksheet 1	Worksheet 2	Worksheet 3	Worksheet 4	



## SESSION 1: Getting Started

# Starting Mathematica

- Menu
- Palettes
- Activating the kernel
- Right-side brackets

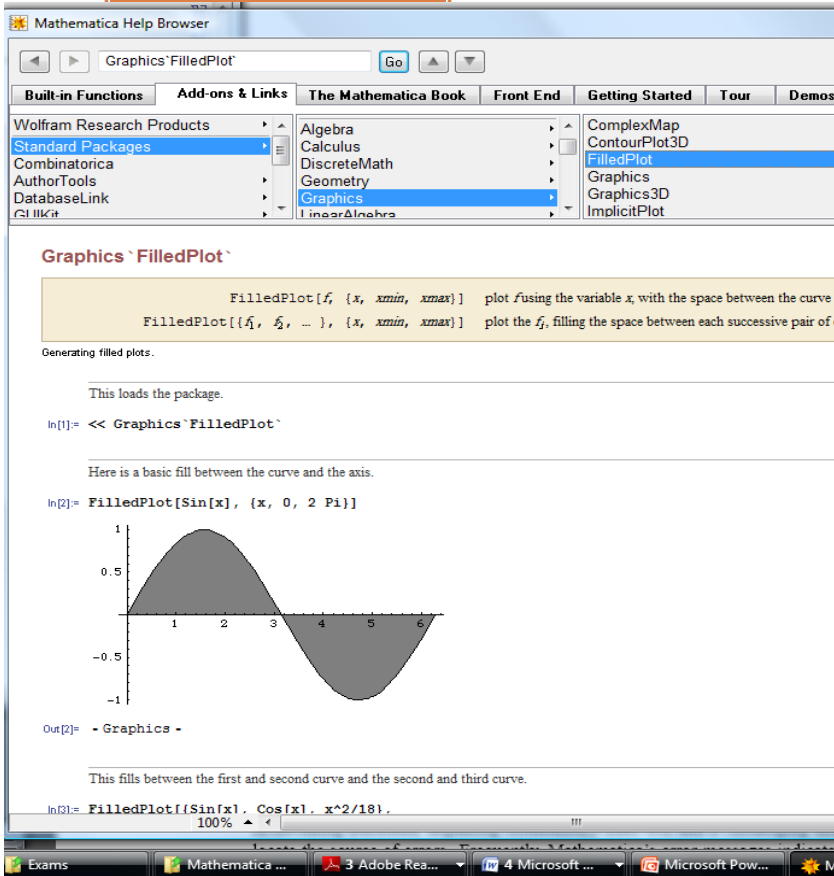


# Mathematica Syntax

## Five Basic Rules of Mathematica Syntax

- The arguments of functions are given in brackets [...]
- Every word of a built-in Mathematica function begins with a capital letter, e.g. Sin[x]
- Multiplication is represented by a \* or space between characters. Enter 2\*x\*y or 2 x y to evaluate 2xy not 2xy.
- Powers are denoted by ^. Enter (8\*x^3)^(1/3) to evaluate  $(8x^3)^{(1/3)} = 8^{1/3}(x^3)^{1/3} = 2x$  instead of  $8x^{1/3}$  which returns  $8x/3$
- Mathematica follows the order of operations exactly. Thus, entering  $(1+x)^{1/x}$  returns  $\frac{(1+x)^1}{x}$  while  $(1+x)^{(1/x)}$  returns  $(1+x)^{1/x}$

# Loading Packages



The screenshot shows the Mathematica Help Browser interface. The search bar contains "Graphics`FilledPlot". The left sidebar shows "Standard Packages" selected. The main content area displays the help text for "Graphics`FilledPlot`".

**Graphics`FilledPlot`**

`FilledPlot[f, {x, xmin, xmax}]` plot f using the variable  $x$  with the space between the curve and the axis filled.

`FilledPlot[{f1, f2, ...}, {x, xmin, xmax}]` plot the  $f_i$ , filling the space between each successive pair of curves.

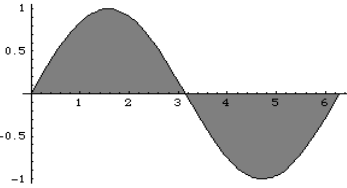
Generating filled plots.

This loads the package.

```
In[1]:= << Graphics`FilledPlot`
```

Here is a basic fill between the curve and the axis.

```
In[2]:= FilledPlot[Sin[x], {x, 0, 2 Pi}]
```



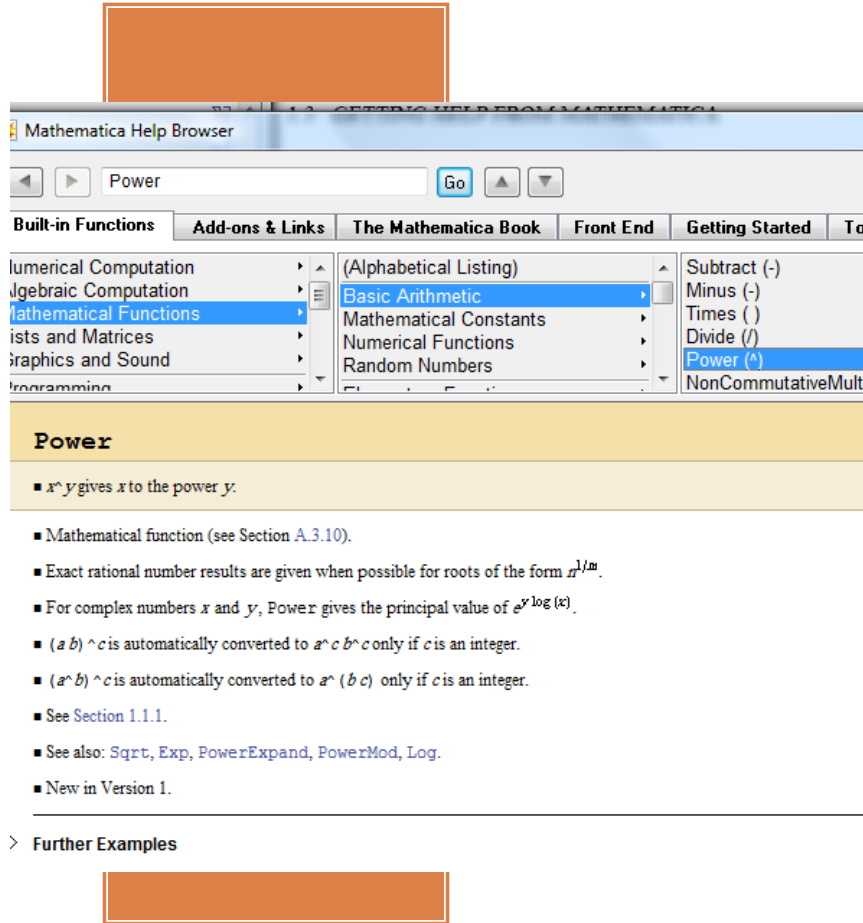
```
Out[2]:= - Graphics -
```

This fills between the first and second curve and the second and third curve.

```
In[3]:= FilledPlot[{Sin[x], Cos[x], x^2/18}, {x, 0, 2 Pi}]
```

- Mathematica contains many built-in functions that are loaded immediately when called.
- Some other functions are contained in packages that must be loaded separately to minimize memory requirements.
- A list of available package groups are in the help facility
- Example, Graphics → FilledPlot

# Getting Help



The screenshot shows the Mathematica Help Browser interface. At the top, there is a search bar with the word "Power" entered and a "Go" button. Below the search bar are several tabs: "Built-in Functions", "Add-ons & Links", "The Mathematica Book", "Front End", "Getting Started", and "To". The "Built-in Functions" tab is active, and a tree view on the left shows "Mathematical Functions" selected. A secondary tree view on the right shows "Basic Arithmetic" selected, with "Power (^)" highlighted. The main content area displays the "Power" function page, which includes a definition:  $x^y$  gives  $x$  to the power  $y$ . Below this are several bullet points: "Mathematical function (see Section A.3.10).", "Exact rational number results are given when possible for roots of the form  $x^{1/m}$ .", "For complex numbers  $x$  and  $y$ , Power gives the principal value of  $e^{y \log(x)}$ .", " $(a b)^c$  is automatically converted to  $a^c b^c$  only if  $c$  is an integer.", " $(a^b)^c$  is automatically converted to  $a^{(b c)}$  only if  $c$  is an integer.", "See Section 1.1.1.", "See also: Sqrt, Exp, PowerExpand, PowerMod, Log.", "New in Version 1." Below the main content area, there is a section titled "Further Examples" with a redacted area below it.

- Becoming competent with Mathematica can take a great time!
- Errors can rise so do not get frustrated!
- Error's messages can indicate where the errors occurred.
- You can access Mathematica's help facility directly from the menu.
- Using ? And ?? → ?Plot, ?Int\*
- Tour of Mathematica

# Numerical Calculations

- Basic arithmetic operations are performed in the natural way
- a plus b  $\rightarrow a+b$
- a minus b  $\rightarrow a-b$
- a times b  $\rightarrow a*b$
- a divided by b  $\rightarrow a/b$
- a raised to the power b  $\rightarrow a^b$



# Numerical Calculations

## Example:

□  $121 + 542$

□  $3231 - 9876$

□  $(-23)(76)$

□  $(22341)(832748)(387281)$

□ 
$$\frac{467}{31}$$

□ 
$$\frac{12315}{35}$$

# Numerical Calculations

- The term  $a^{n/m}$  when  $n/m = 1/2$  we can use the command `Sqrt[a]`.
  - Example: Compute
    1.  $\sqrt{27}$
    2. Use `N[...]`
    3.  $(-27/64)^{(2/3)}$
    4. Use `N[...]`
    5. Repeat 3 by hand
- <<Miscellaneous 'RealOnly'

# Built-in Constants

- Mathematica has built-in definitions of commonly used constants.
  - $e \approx 2.71828 \rightarrow E$
  - $\pi \approx 3.14159 \rightarrow \text{Pi}$
  - $i = \sqrt{-1} \rightarrow I$ , complex arithmetic is done automatically.  $\text{Re}[z]$ ,  $\text{Im}[z]$ ,  $\text{Conjugate}[z]$ ,  $\text{Abs}[z]$ ,  $\text{Arg}[z]$
- Example: Compute
  - $N[e, 50]$
  - $(3+i) / (4-i)$

# Built-in Functions

- Mathematica contains numerous mathematical functions.
  - $\text{Exp}[x]$  : exponential function
  - $\text{Log}[x]$  : natural logarithm
  - $\text{Abs}[x]$  : absolute value
  - $\text{Sin}[x]$ ,  $\text{Cos}[x]$ ,  $\text{Tan}[x]$ ,  $\text{Sec}[x]$ ,  $\text{Csc}[x]$  and  $\text{Cot}[x]$  : trigonometric functions
  - $\text{ArcSin}[x]$ ,  $\text{ArcCos}[x]$ ,  $\text{ArcTan}[x]$ ,  $\text{ArcSec}[x]$ ,  $\text{ArcCsc}[x]$  and  $\text{ArcCot}[x]$  : inverse trigonometric functions
  - $\text{Sinh}[x]$ ,  $\text{Cosh}[x]$  and  $\text{Tanh}[x]$  : hyperbolic trigonometric functions

# Built-in Functions

- Example: Compute
  - `N[Exp[-5]]`
  - `Log[Exp[x]]`
  - `Exp[Log[4]]`
  - `Abs[- $\pi$ ]`
  - `Sin[ $\pi/12$ ]` → add `N[...]`

# Built-in Functions

- Example: What do these functions do? If you don't know how can you find out?
  - TrigExpand
  - TrigReduce
  - ExpToTrig
  - TrigToExp
  - Simplify

# Built-in Functions

- Example: Compute
  - $\text{Cos}[x]^2 + \text{Sin}[x]^2$
  - Simplify the last expression
  - `TrigExpand[Cos[3x]]`
  - `TrigReduce[Sin[3x] Cos[4x]]`
  - `ExpToTrig[1/2 (Exp[x]+Exp[-x])]`
  - `TrigToExp[Sin[x]]`

# Word Processing in Notebooks

- You can set the following cell options from the Format menu
  - Font
  - Size
  - Text color
  - Background color
  - Word wrapping
  - Justification
  - Style
  - Text alignment
- Check spelling can be found under Edit
- Inserting a formula in a text cell (Press CTRL 9) then use keystrokes
- Copy and paste between cells as usual



# Word Processing in Notebooks

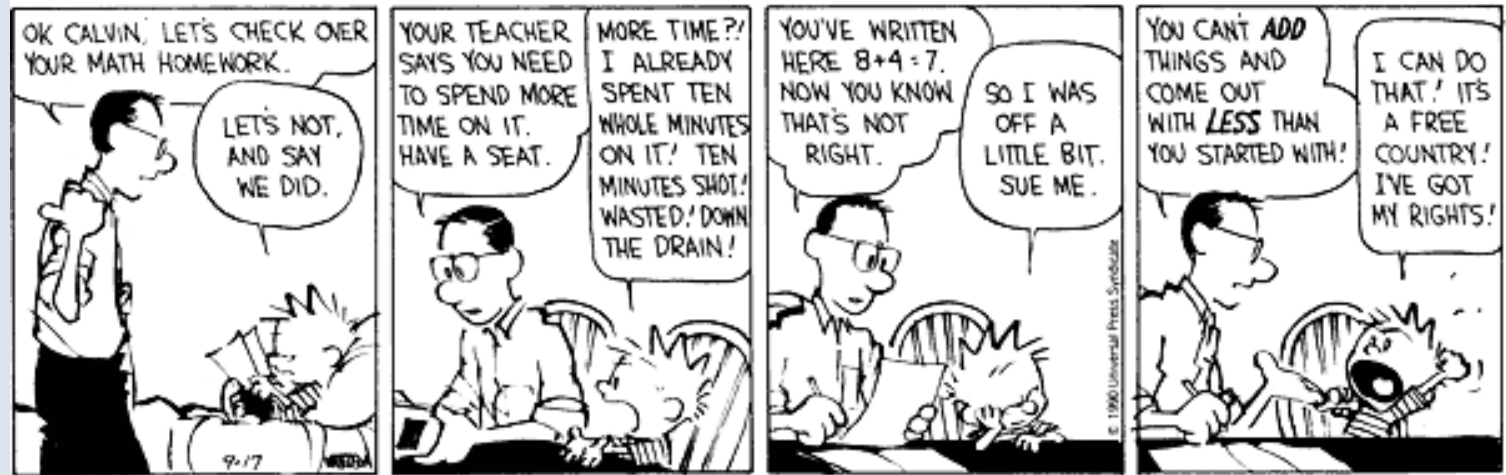


## Example:

- ▣ Change the background color of your cell
- ▣ Change the content of a cell to text style
- ▣ Insert a formula into a text cell

# Work Session

Please solve the question given to you in worksheet 1



## SESSION 2: User Defined Functions and Plotting

# Variables, Equations and Functions

- To define a variable:
  - ▣  $x = a$  (begins with a small letter)
- To define an equation:
  - ▣  $eq == a ; x + 5 == b$
- To define a function:
  - ▣  $f[x_] := \text{expression in } x$
  - ▣ Evaluate a function:  $f[a]$

# Plotting Functions

- To plot a function:
  - ▣ `Plot[f[x],{x,a,b}]`
- Example:
  - ▣ Expand  $(2x+1)(3x-1)(x-1)$
  - ▣ Define a function  $f(x)$  and make its form the previous expanded terms
  - ▣ Plot the function  $f(x)$  with  $-1 << x << 3/2$

# Solving Equations and Inequalities

- To solve an equation we use:
  - ▣ `Solve[lhs==rhs,x]`
- To solve a system of equations:
  - ▣ `Solve[{system of equations},{variables}]`
- To solve an inequality (load package):
  - ▣ `InequalitySolve[inequality,x]`
- Solving equation by finding the root:
  - ▣ `FindRoot[equation of x, {x,a}]`

# Solving Equations and Inequalities

## □ Example:

□ Solve the equation  $x^2 - 1 = 0$

□ Solve the inequality  $4 - x^2 \geq 0$

□ Solve the system of equations

$$x = y - 1$$

$$x^2 = 2y + 6$$

□ Solve  $eq = x^2 + bx + 1$  then substitute  $b=9$ , you should get two roots.

# More Plotting

- To graph the parametric equations

$$x = x(t), y = y(t), a \leq t \leq b$$

we use:

```
ParametricPlot[{x[t],y[t]},{t,a,b}]
```

- To graph a polar function

$$r = r(\theta), \alpha \leq \theta \leq \beta$$

we use:

```
PolarPlot[r[theta],{theta,alpha,beta}]
```



# More Plotting

- 3D plotting we use:

`Plot3D[f[x,y],{x,a,b},{y,c,d}]`

- We can also do a contour plot using

`ContourPlot[f[x,y],{x,a,b},{y,c,d}]`

# More Plotting

- Example:

- Graph the parametric equation

$$x = \text{Cos}(5t), y = \text{Sin}(3t), 0 \leq t \leq 2\pi$$

- Graph a polar function

$$r = \text{Cos}(8\theta / 3), 0 \leq \theta \leq 6\pi$$

- Do a 3D plot and a contour plot of

$$\text{Sin}(xy), 0 \leq x \leq 4, 0 \leq y \leq 4$$

# Work Session

Please solve the question given to you in worksheet 2

# Happy Face Math

$$\text{😊}^{-1} = \text{😬}$$

$$\text{😊}^2 = \text{😊}$$

$$\text{😊}^3 = \text{😊}$$

$$\text{sup}(\text{😊}) = \text{😊}$$

$$\partial(\text{😊}) = \text{😊}$$

$$\text{sin}(\text{😊}) = \text{😊}$$

$$\text{Re}(\text{😊}) = \text{😊} \text{ No i's}$$

$$\text{Im}(\text{😊}) = \dots$$

$$\nabla \times (\text{😊}) = \text{😊}$$

$$\nabla(\text{😊}) = \text{😊}$$

$$\text{log}(\text{😊}) = \text{😊}$$

Happy Face Math by Charlie Smith

## SESSION 3: Calculus

# Limits...

- For finding limits we use:

Limit[f[x],x->a]

- ▣ a can be +/- infinity just write Infinity

- Example:

- ▣ Plot the function  $\frac{\sin 3x}{x}$  then find its limit when  $x \rightarrow 0$

# Derivatives...

- We can find the derivative by using its definition:

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- Or by using the function D and ‘:  
D[f[x],x] , f'[x], f''[x], D[f[x],{x,n}]  
D[f[x,y],{x,n},{y,m}]

- Example:

- Write the following command

$$D[f[x] g[x],x]$$

- Compute the first and second derivative of:

$$f(x) = 4x^5 - \frac{5}{2}x^4 - 10x^3$$

# Integration...

- We can integrate a function by using:  
`Integrate[f[x],x]`
- We can also perform definite integration  
`Inegrate[f[x],{x,a,b}]`  
`Integrate[f[x,y],{y,ymin,ymax},{x,xmin,xmax}]`
- When we can't get an exact solution of the integration, we can use a numerical approximation to the integral  
`NIntegrate[f[x],{x,a,b}]`

- Example:

- ▣ Evaluate

$$\int_1^4 (2x^2 + 1) / \sqrt{x} dx$$

# Work Session

Please solve the question given to you in worksheet 3



PETER

1.21

4b) Expanded

$$(a+b)^n$$

$$= (a + b)^n$$

$$= (a + b)^n$$

$$= (a + b)^n$$

etc...

*Very funny, Peter*

## SESSION 4: Series, Sums and Matrices

# Sums and Series...

- We can obtain the sum of a series by writing:  
 $\text{Sum}[a[k],\{k,1,\infty\}]$
- We can find the Maclaurin series or the Taylor series using:  
 $\text{Series}[f[x],\{x,x_0,n\}]$
- Example:
  - ▣ Execute  $\text{Series}[\text{Cos}[x],\{x,0,4\}]$

# Vectors and Matrices...

- We can define a vector as:  
 $r = \{x, y, z\}$  ,  $r1 = \{x1, y1, z1\}$
- Operations on vectors:  
 $r - r1$   
 $a r \leftarrow$  scalar multiplied by a vector  
 $r.r1 \leftarrow$  dot product  
 $\text{Cross}[r, r1] \leftarrow$  cross product
- $\text{Array}[v, n] \leftarrow$  n-dimensional vector
- $\text{Table}[f[k], \{k, n\}]$
- $\{a, b\} = \text{Take}[\{l, m, n\}, 2]$

# Vectors and Matrices...

- `MatrixForm[name]` ← output form
- `Det[A]` ← determinant of a matrix
- `Inverse[A]` ← inverse of a matrix
- `Transpose[A]` ← transpose of a matrix
- `Part[A,l,j]` ← gives the element  $A_{ij}$
- `IdentityMatrix[n]` for an  $n \times n$  matrix
- `DiagonalMatrix[list]`,  $list = \{a,b,c,d,\dots\}$

# Work Session

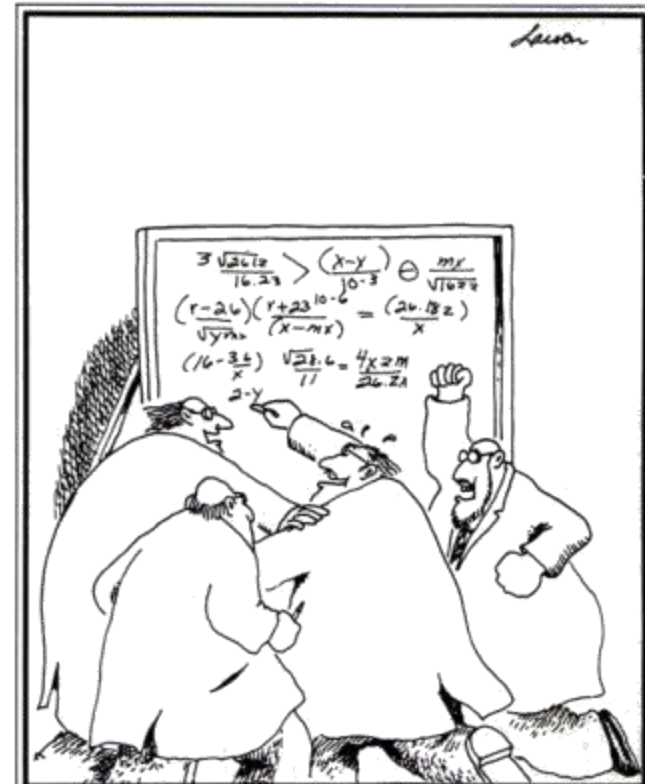
Please solve the question given to you in worksheet 4

# Some Examples in Physics

- Interference and diffraction from multiple slits
- Fourier transform

# Notes...

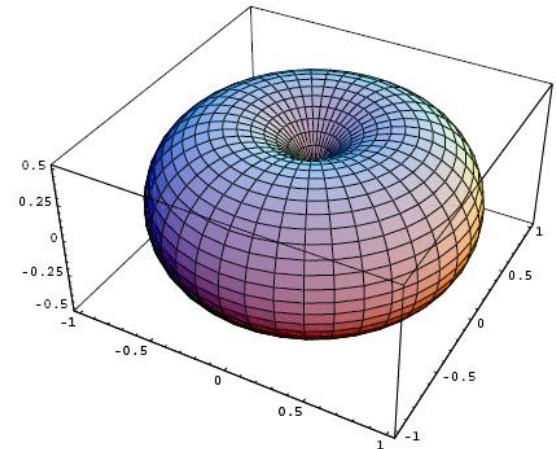
- Clear variable before using them! Clear[x] or x=.
- Don't say I don't know!! Use ? or ??
- /. Frustration → fun



"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"

# So much more...

- Differential equations
- Animations
- Fitting data
- Programming
- Iterations (Do, For, While)
- Visit [www.wolfram.com](http://www.wolfram.com)





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"Are you sure you're not assigning too much homework? — I don't want to end up *overqualified*."

## Your Project...

Must be handed in by the end of the second week of next semester

**Wednesday 23/3/1432 H**

Submission through email: [raltuwirqi@kau.edu.sa](mailto:raltuwirqi@kau.edu.sa)