WAVE PROPERTIES OF PARTICLES

1. De Broglie waves.
2. Waves of what?
3. Describing a wave.
4. Phase and group velocities.
5. Particle diffraction.
6. Particle in a box.
7. Uncertainty principle I.
8. Uncertainty principle II.
9. Applying the uncertainty principle.

DESCRIBING A WAVE

How fast do de Broglie waves travel?

This wave has the same velocity as the body it is associated with.

Let us call the de Broglie wave velocity υ_p then:

$$\upsilon_p = \nu\lambda$$

$$E = h\nu = \gamma mc^2$$

$$\lambda = \frac{h}{\gamma m\upsilon}$$

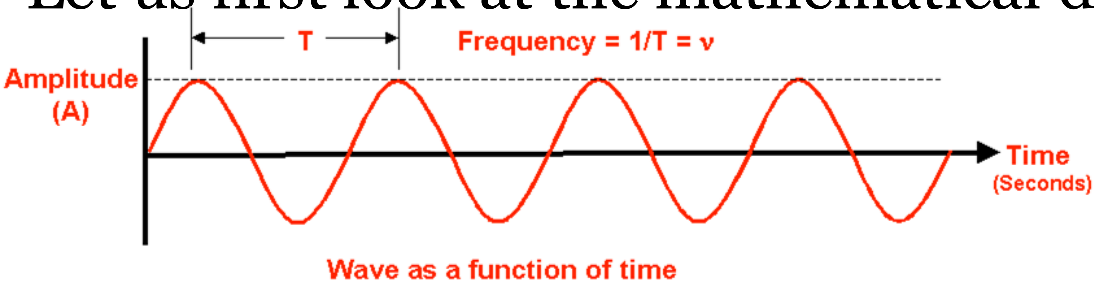
$$\upsilon_p = \nu\lambda = \left(\frac{\gamma mc^2}{h}\right)\left(\frac{h}{\gamma m\upsilon}\right) = \frac{c^2}{\upsilon}$$

Faster than light!!

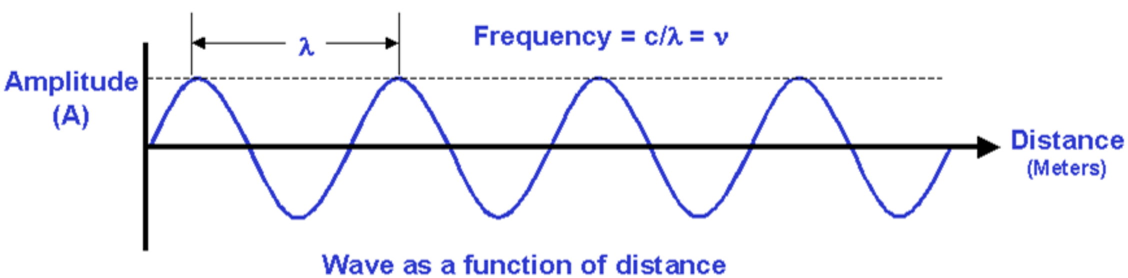
DESCRIBING A WAVE

To resolve this we need to distinguish between **phase velocity** and **group velocity**.

Let us first look at the mathematical description of waves.



$$y = A\cos 2\pi\nu t$$



$$y = A\cos\frac{2\pi}{\lambda}x$$

$$y = A\cos(\omega t - kx)$$

ω angular frequency

$$k \text{ wave number} = \frac{\omega}{\upsilon_p}$$

$$y = A\cos 2\pi\left(\nu t - \frac{x}{\lambda}\right)$$

$$\upsilon_p = \nu\lambda$$

DESCRIBING A WAVE

Remember…

The general formula of a wave is:

$$y = A\cos(\omega t - kx)$$

ω angular frequency

$$k \text{ wave number} = \frac{\omega}{\upsilon_p}$$

PHASE AND GROUP VELOCITIES

De Broglie waves…

→ the probability of finding a moving body at a particular place at a particular time.

Can we represent the de Broglie wave by a simple wave equation?

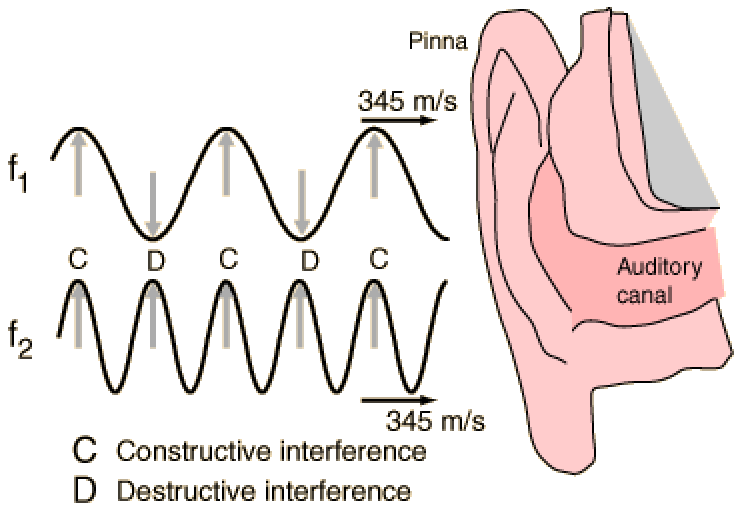
→No! Simple wave formula of an indefinite series of waves all with the same amplitude A. $y = A\cos(\omega t - kx)$

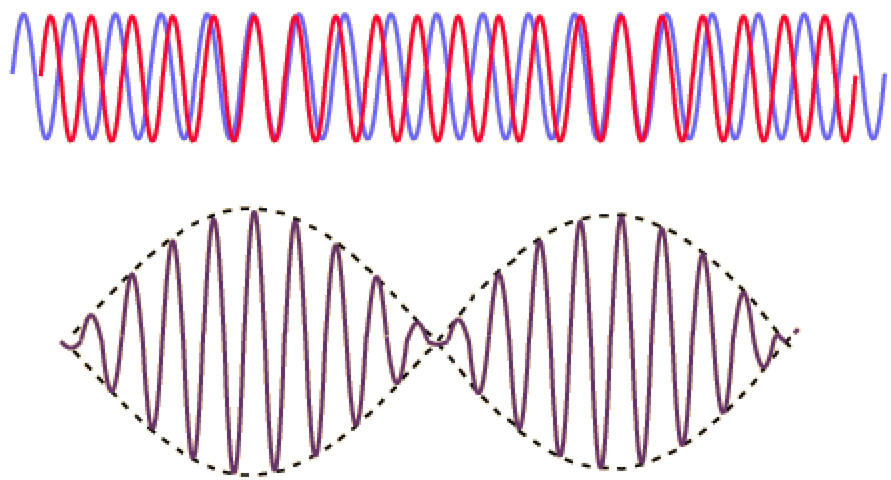
→ A wave representing a moving body must be a **wave packet** or **wave group**.

PHASE AND GROUP VELOCITIES

Wave packet or wave group…

A familiar example is **beats**.

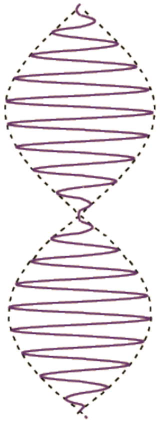


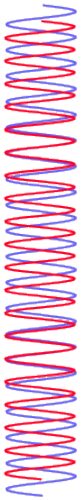


How can we describe this mathematically?

SUPERPOSITION!

PHASE AND GROUP VELOCITIES





Superposition of what?

Individual waves of different wavelengths whose interference with one another results in a variation in amplitude that defines the group shape.

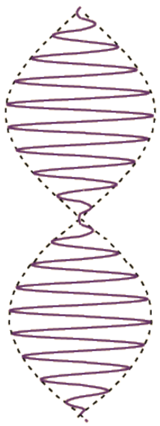
→If the velocities of the waves are the same, the velocity of the wave group is the common phase velocity.

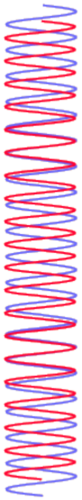
→ If the phase velocity varies with wavelength, the individual waves do not proceed together. ← **dispersion**!

→ the wave group has a velocity different from the phase velocities of the waves that make it up. ← de Broglie waves.

PHASE AND GROUP VELOCITIES

How can we calculate υ_g?





Let us assume that the wave group arises from the combination of two waves that have the same amplitude *A* but differ by an amount $\Delta\omega$ in angular frequency and Δk in wave number.

$$y_1 = A\cos(\omega t - kx)$$

$$y_2 = A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

$$y = y_1 + y_2$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$y = y_1 + y_2 = 2A\cos\frac{1}{2}[(2\omega + \Delta\omega)t - (2k + \Delta k)x]\cos\frac{1}{2}(\Delta\omega t - \Delta kx)$$

PHASE AND GROUP VELOCITIES

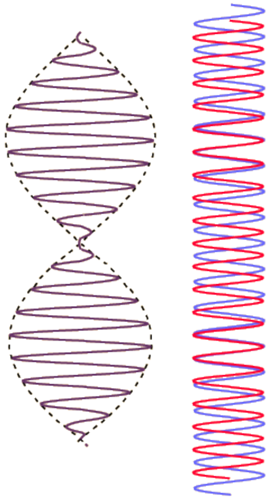
$$y = y_1 + y_2 = 2A\cos\frac{1}{2}\left[(2\omega+\Delta\omega)t-(2k+\Delta k)x\right]\cos\frac{1}{2}(\Delta\omega t-\Delta kx)$$

$\Delta\omega$ and Δk are small compared with ω and k.

$$2\omega+\Delta\omega \approx 2\omega$$

$$2k+\Delta k \approx 2k$$

$$y = 2A\cos(\omega t-kx)\cos\left(\frac{\Delta\omega}{2}t-\frac{\Delta k}{2}x\right)$$



This is a wave of angular frequency ω and wave number k that has superimposed upon it a modulation of angular frequency $\Delta\omega/2$ and wave number $\Delta k/2$.

$$\upsilon_p = \frac{\omega}{k}$$

$$\upsilon_g = \frac{\Delta\omega}{\Delta k}$$

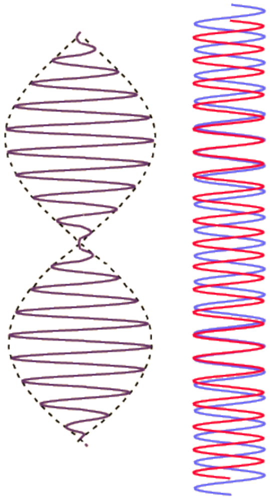
$$\upsilon_g = \frac{d\omega}{dk}$$

PHASE AND GROUP VELOCITIES

For a de Broglie wave associated with a body of mass *m* moving with velocity υ:

The angular frequency:

$$\omega = 2\pi\nu = \frac{2\pi\gamma mc^2}{h} = \frac{2\pi\gamma mc^2}{h\sqrt{1-\upsilon^2/c^2}}$$



The wave number:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\gamma m\upsilon}{h} = \frac{2\pi m\upsilon}{h\sqrt{1-\upsilon^2/c^2}}$$

Both ω and *k* are functions of the body's velocity υ.

What is the group velocity of de Broglie waves?

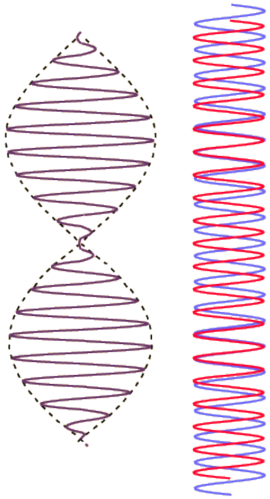
PHASE AND GROUP VELOCITIES

What is the group velocity of de Broglie waves?

For a de Broglie wave associated with a body of mass

$$\upsilon_g = \frac{d\omega}{dk} = \frac{d\omega / d\upsilon}{dk / d\upsilon}$$

$$\frac{d\omega}{d\upsilon} = \frac{2\pi m\upsilon}{h(1-\upsilon^2/c^2)^{3/2}} \qquad \frac{dk}{d\upsilon} = \frac{2\pi m}{h(1-\upsilon^2/c^2)^{3/2}}$$



$$\upsilon_g = \upsilon$$

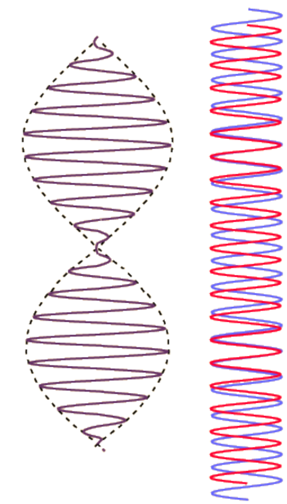
The de Broglie wave group associated with a moving body travels with the same velocity as the body.

PHASE AND GROUP VELOCITIES

What is the phase velocity of de Broglie waves?

$$\upsilon_p = \frac{\omega}{k} = \frac{c^2}{\upsilon}$$

It exceeds both the velocity of the body and the velocity of light.



How can this be justified?

υ_p has no physical significance because the motion of the group not the individual correspond to the motion of the body and $\upsilon_g < c$.

→ De Broglie wave does not violate special relativity!

PHASE AND GROUP VELOCITIES

Example 3.3:

An electron has a de Broglie wavelength of 2.00 pm = 2.00×10^{-12} m. Find its kinetic energy and the phase and group velocities of it de Broglie waves.