RELATIVITY

1. Special Relativity
2. Time Dilation
3. Doppler Effect
4. Length Contraction
5. Twin Paradox
6. Electricity and Magnetism
7. Relativistic Momentum
8. Mass and Energy
9. Energy and Momentum
10. General Relativity

ENERGY AND MOMENTUM

What do we know so far?

-The total energy and momentum are conserved in an isolated system.
- The rest energy of a particle is invariant.

Hence, the E and p and E_o are **more fundamental** than velocity and KE.

How are these three quantities related?

ENERGY AND MOMENTUM

Linking *E* and *p* and E_o ...

Let us start with the equation of total energy and square it:

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-\upsilon^2/c^2}} \Rightarrow E^2 = \frac{m^2c^4}{1-\upsilon^2/c^2}$$

And for the momentum, square it and × c²:

$$p = \gamma m\upsilon = \frac{m\upsilon}{\sqrt{1-\upsilon^2/c^2}} \Rightarrow p^2c^2 = \frac{m^2\upsilon^2c^2}{1-\upsilon^2/c^2}$$

Subtract p^2c^2 from E^2:

$$E^2 - p^2c^2 = \frac{m^2c^4 - m^2\upsilon^2c^4}{1-\upsilon^2/c^2} = \frac{m^2c^4(1-\upsilon^2/c^2)}{1-\upsilon^2/c^2} = (mc^2)^2$$

$$E^2 = (mc^2)^2 + p^2c^2$$

ENERGY AND MOMENTUM

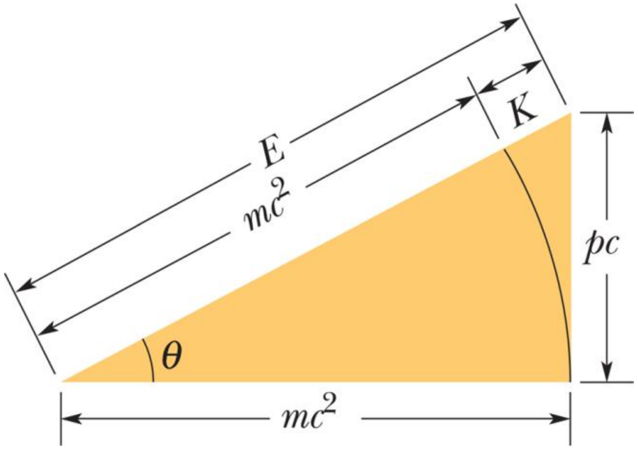
Linking *E* and *p* and E_o ...

$$E^2 = (m c^2)^2 + p^2 c^2$$

Since mc^2 is invariant, then so is

$E^2 - p^2c^2$ is invariant:

this quantity for a particle has the same value in all frames of reference.



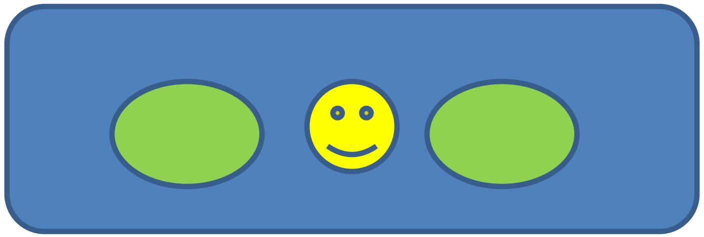
ENERGY AND MOMENTUM

What if we had a system of particles?

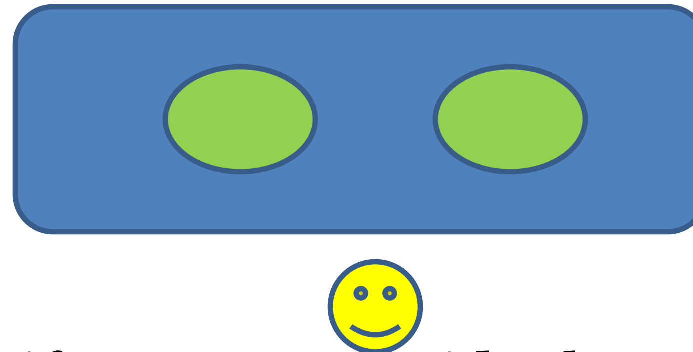
$$E^2 = (m c^2)^2 + p^2 c^2$$

- The rest energy – hence mass m – is that of the ***entire*** system.
- If the particles in the system are moving with respect to one another, the sum of their individual rest energies may not equal the rest energy of the system. ←Ex. 1-7.

ENERGY AND MOMENTUM



if we were inside the system then 0.5 kg of mass is the conversion into KE of the smaller bodies.



if we were outside the system, then the system is at rest before and after the explosion, so the system didn't gain KE.
The rest energy of the system includes the KE of its internal motions and corresponds to mass 2.5 kg both before and after the explosion.

ENERGY AND MOMENTUM

Can the rest energy of the entire system be less than its constituents?

YES! In the case that the members of the system of particles are held together by attractive forces. (ex. Neutron and proton).

The difference in energy is called binding energy of the nucleus. ← the least amount of energy needed to break it up.

Nuclear binding energy of a nucleus ~ 10^{12} kJ/kg. For water it is 2260 kJ/kg.

ENERGY AND MOMENTUM

Can a massless particle exist?
Can a particle exist which has no rest mass but exhibits particle-like properties like energy and momentum?

In classical mechanics NO! In relativistic mechanics YES!

$$E = \gamma m c^2 = \frac{m c^2}{\sqrt{1 - \upsilon^2 / c^2}}$$

$$p = \gamma m \upsilon = \frac{m \upsilon}{\sqrt{1 - \upsilon^2 / c^2}}$$

When $m = 0$ and $\upsilon << c$ then $E = p = 0$
When $m = 0$ and $\upsilon = c$ then $E = 0/0$ and $p = 0/0$ ← indeterminate: E and p can have any values.

A massless particle that possess energy and momentum exists **provided that they travel with the speed of light**.

$$E^2 = (m c^2)^2 + p^2 c^2 \quad \longrightarrow \quad E = pc$$

Photons

ENERGY AND MOMENTUM

What is the unit for measuring energy?

In atomic physics the usual unit is **electronvolt (eV)**

1 eV is the energy gained by an electron accelerated through a potential difference of 1 V.

$$1\,eV = (1.602\times 10^{-19}\ C) + (1.0\ \mathrm{V}) = 1.602\times 10^{-19}\ J$$

What do we use eV to measure?

- Ionization energy of an atom: work needed to remove one of its electrons. (for N_2: 14.5 eV)

-Binding energy of a molecule: energy needed to break it apart into separate atoms. (for H_2: 4.5 eV)

- Higher energies in the atomic realm are expressed in **kiloelctronvolts** (1 keV = 10^3 eV)

ENERGY AND MOMENTUM

What do we use eV to measure?

-In nuclear and elementary-particle physics the higher energies are measured in **megaelectronvolts** (MeV = 10^6 eV) and **gigaelectronvolts** (GeV = 10^9 eV).

- fission events releases ~ 200 MeV.

So what are the units used for energy and momentum?

E → eV

p → eV/c

Rest mass → eV/c^2

$$E^2 = (m c^2)^2 + p^2 c^2$$

ENERGY AND MOMENTUM

Remember…

Energy and momentum are connected in relativity through

$$E^2=(m\,c^2)^2 + \mathrm{p}^2\,\mathrm{c}^2$$

ENERGY AND MOMENTUM

Example 1.8:

An electron ($m = 0.511\ MeV/c^2$) and a photon ($m=0$) both have momenta of 2.0 MeV/c. Find the total energy of each.