RELATIVITY

1. Special Relativity
2. Time Dilation
3. Doppler Effect
4. Length Contraction
5. Twin Paradox
6. Electricity and Magnetism
7. Relativistic Momentum
8. Mass and Energy
9. Energy and Momentum
10. General Relativity

LENGTH CONTRACTION

- Measurement of length is *affected by relative motion*.
- The length L of an object in motion with respect to an observer always appear to the observer to be **shorter** than its length L_o when it is at rest with respect to him.
- The **contraction** occurs only **in the direction of the relative motion**.
- The length L_o of an object in its rest frame is called its **proper length**.
- (Sam's and Sally's example, clock moving *perpendicular* to $\mathbf{v} \rightarrow L = L_o$.

LENGTH CONTRACTION

How can we derive the length contraction?

- The simplest way is based on time dilation and the principle of relativity.
- We will consider the example of muons.

LENGTH CONTRACTION

What is a muon?

- Unstable particle created at high altitudes by fast cosmic-ray particles (largely protons) from space when they collide with atomic nuclei in the earth's atmosphere.
- A muon has a mass 207 times that of the electron and has a charge of either $+e$ or $-e$.
- It decays into an electron or a positron after an average lifetime of 2.2 μs (2.2×10^{-6} s).
- Cosmic-ray muons have speed of about 2.994×10^{8} m/s ($0.998c$).

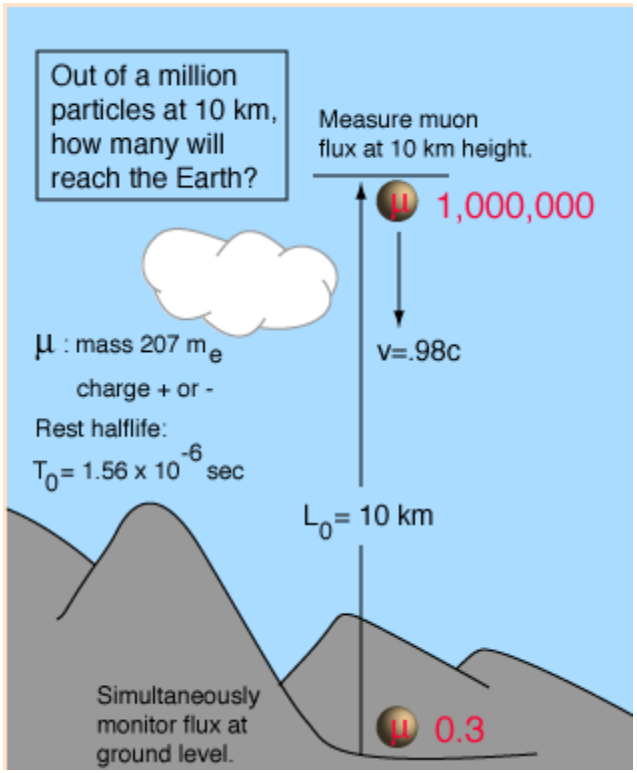
How far can a muon travel?

LENGTH CONTRACTION

How far can a muon travel?

- In its average lifetime of t_o = 2.2 the muon can before decaying travel a distance…

$\upsilon\, t_o = (2.994 \times 10^8 \text{ m/s})(2.2 \times 10^{-6} \text{ s}) = 6.6 \times 10^2 \text{ m} = 0.66 \text{ km}$



LENGTH CONTRACTION

Let us think for a minute!!

What is the reference frame that we are doing our measurement in???

LENGTH CONTRACTION

How far can a muon travel?

- Average lifetime of t_o = 2.2 the muon is according to an observer at rest with respect to the muon.
- The muon's life time in our frame of reference is extended by ***time dilation***.

$$t = \frac{t_o}{\sqrt{1-\upsilon^2/c^2}} = \frac{2.2\times10^{-6}\,s}{\sqrt{1-(0.998c)^2/c^2}} = 34.8\times10^{-6}\,s = 34.8\,\mu s$$

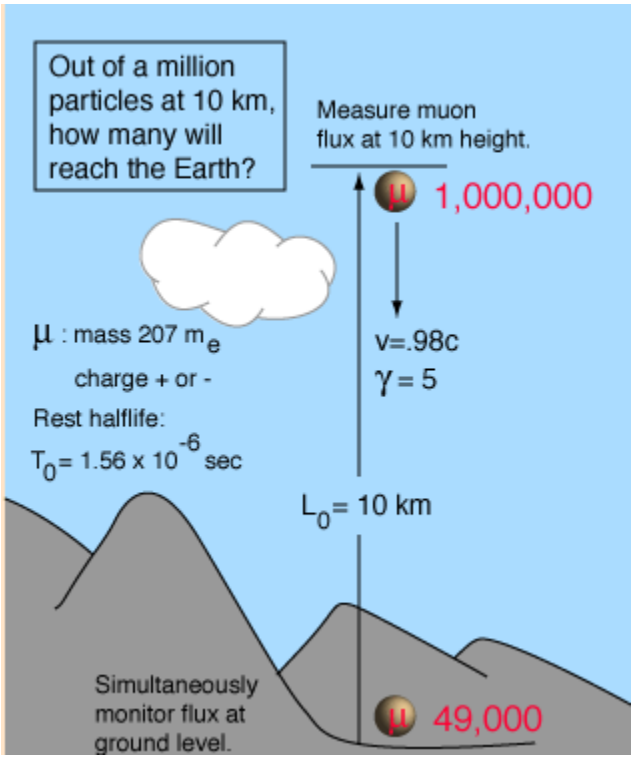
- The moving muons have life times almost 16 times longer than those at rest.
- In a time interval of 34.8 μs, a muon whose speed is 0.998c can cover a distance..

$\upsilon\, t = (2.994 \times 10^8 \text{ m/s})(34.8 \times 10^{-6} \text{ s}) = 1.04\times10^4 \text{ m} = 10.4 \text{ km}$

LENGTH CONTRACTION

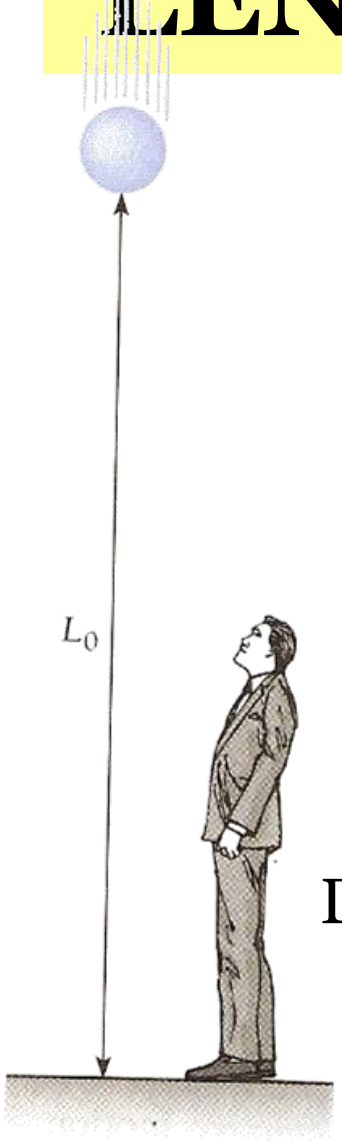
How far can a muon travel?

- A muon can reach ground from altitudes of as much as 10.4 km because ***in the frame*** in which these altitudes are ***measured***, the muon lifetime is t = 34.8 μs.

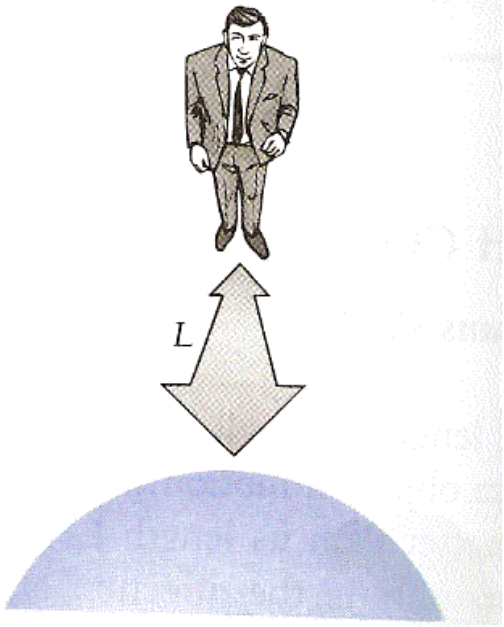


LENGTH CONTRACTION

muon lifetime is t_o = 2.2 μs.
Distance covered L = 0.66 km



muon lifetime is t = 34.8 μs.
Distance covered L_o = 10.4 km



From the principle of relativity: $\sqrt{1-v^2/c^2}$

the extent of shortening ≡ the extent extending

LENGTH CONTRACTION

From the principle of relativity:
the extent of shortening ≡ the extent extending

$$\sqrt{1-v^2/c^2}$$

Time dilation

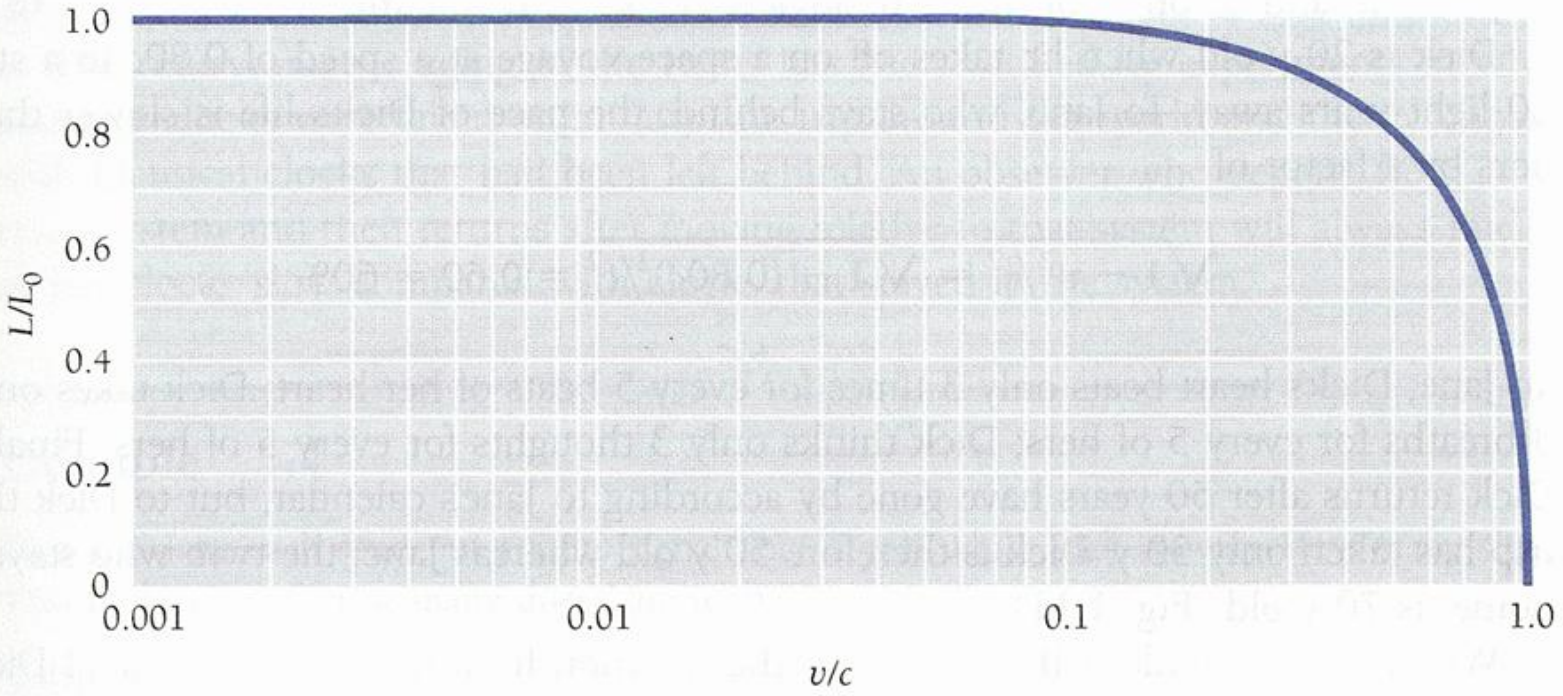
$$t = \frac{t_o}{\sqrt{1-v^2/c^2}}$$

Length contraction

$$L = L_o\sqrt{1-v^2/c^2}$$

LENGTH CONTRACTION

When is the length contraction significant?



LENGTH CONTRACTION

- To a person in a spacecraft, objects on the earth appear shorter than they did when he or she was on the ground by the same factor that the spacecraft appears shorter to somebody at rest.
- The proper length L_o found in the rest frame is the maximum length any observer will measure.
- Length contraction occurs only in the direction of motion.

LENGTH CONTRACTION

Remember…

Faster means shorter.