RELATIVITY

1. Special Relativity
2. Time Dilation
3. Doppler Effect
4. Length Contraction
5. Twin Paradox
6. Electricity and Magnetism
7. Relativistic Momentum
8. Mass and Energy
9. Energy and Momentum
10. General Relativity

RELATIVISTIC MOMENTUM

What is momentum?

- $\mathbf{p} = m\,\mathbf{v}$

Why is momentum an important quantity?

- Linear momentum is conserved in a system of particles not acted upon by outside forces.
- The vector sum of the momenta of particles before the event is equal to their vector sum afterward.

RELATIVISTIC MOMENTUM

Is $\mathbf{p} = m\,\mathbf{v}$ a valid definition of momentum in inertial frames in relative motion?

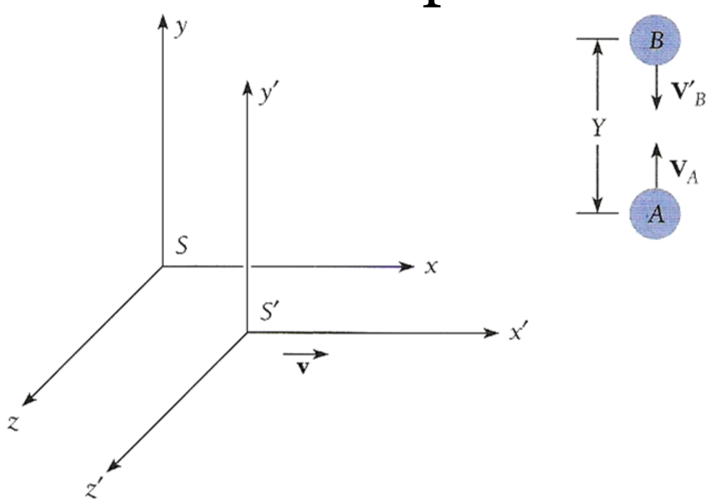
If the answer is no, what is the relativistically correct definition?

- **p** is conserved for all observers in relative motion at constant velocity.
- $\mathbf{p} = m\,\mathbf{v}$ holds in classical mechanics for $v << c$.
- relativistic **p** must be reduced to m**v** for low velocities.

RELATIVISTIC MOMENTUM

Let us start by considering an elastic collision (kinetic energy is conserved) between two particles A and B, as witnessed by observers in the reference frames S and S' which are in uniform relative motion…

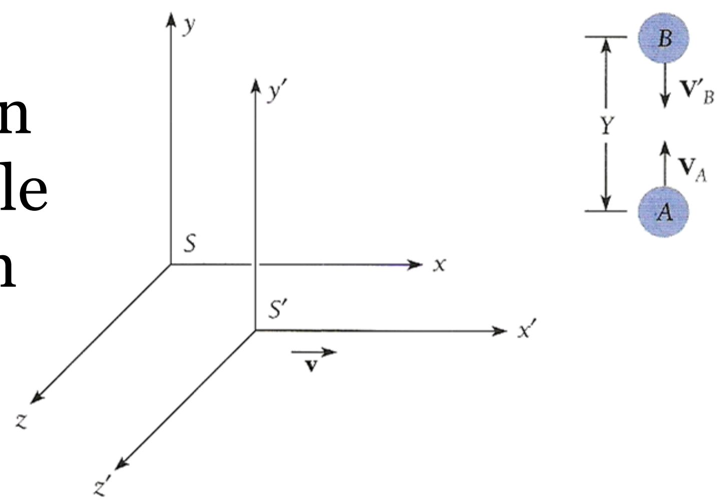
- The properties of A and B are identical when determined in reference frames in which they are at rest.
- Frame S' is moving in the +x direction with respect to S at the velocity **v**.



RELATIVISTIC MOMENTUM

Before collision…

- Particle A is at rest in frame S and particle B at rest in frame S'.
- At the same instant, A was thrown in the +y direction with speed of V_A while B was thrown in the –y direction with speed V_B'.
- where $V_A = V_B$'.
- The behavior of A as seen from S is exactly the same as the behavior of B as seen from S'.



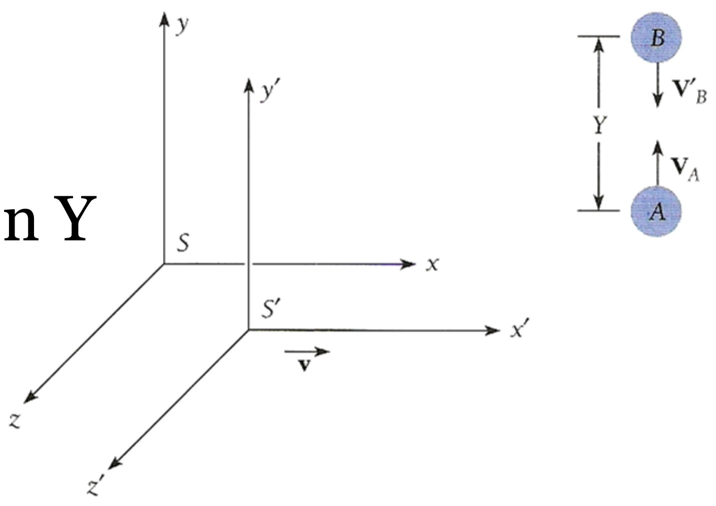
RELATIVISTIC MOMENTUM

During the collision…

- A rebound in the –y direction with speed V_A.
- B rebound in the +y direction with speed V'_B.
- If the particles thrown from position Y apart→

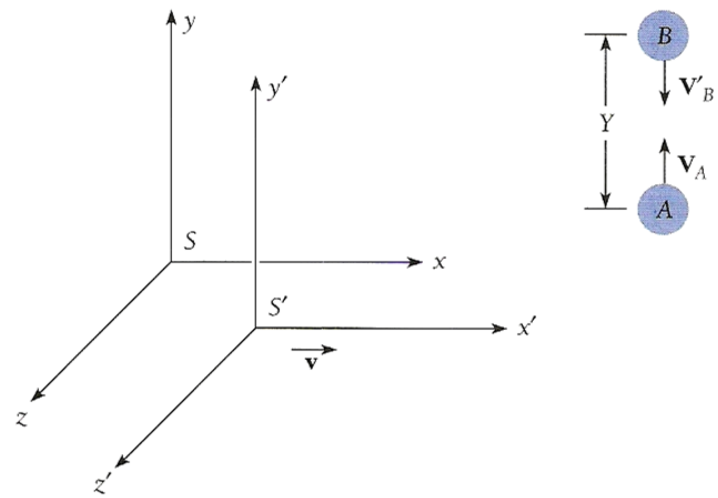
observer in S find that the collision occurs at y = ½ Y.

observer in S' find that it occurs in y' = y = ½ Y.



RELATIVISTIC MOMENTUM

During the collision…
What is the round-trip time measured in both frames?



-The round-trip time T_o for A as measured in frame S is:

$$T_o = \frac{Y}{V_A}$$

- The round-trip time T_o for B as measured in S' is:

$$T_o = \frac{Y}{V_B^{'}}$$

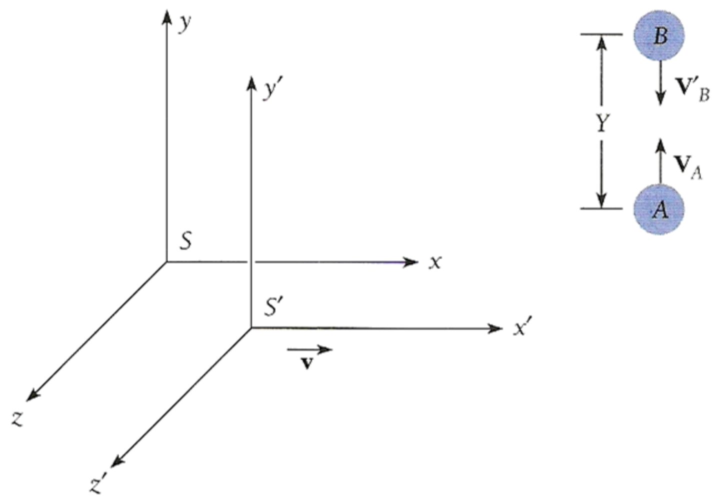
RELATIVISTIC MOMENTUM

During the collision…

- From S the speed V'_B is: $V_B = \dfrac{Y}{T}$

- where T is the time required for B to make its round-trip as measured in S.
- From S' the time for B to make its round-trip is T_o.

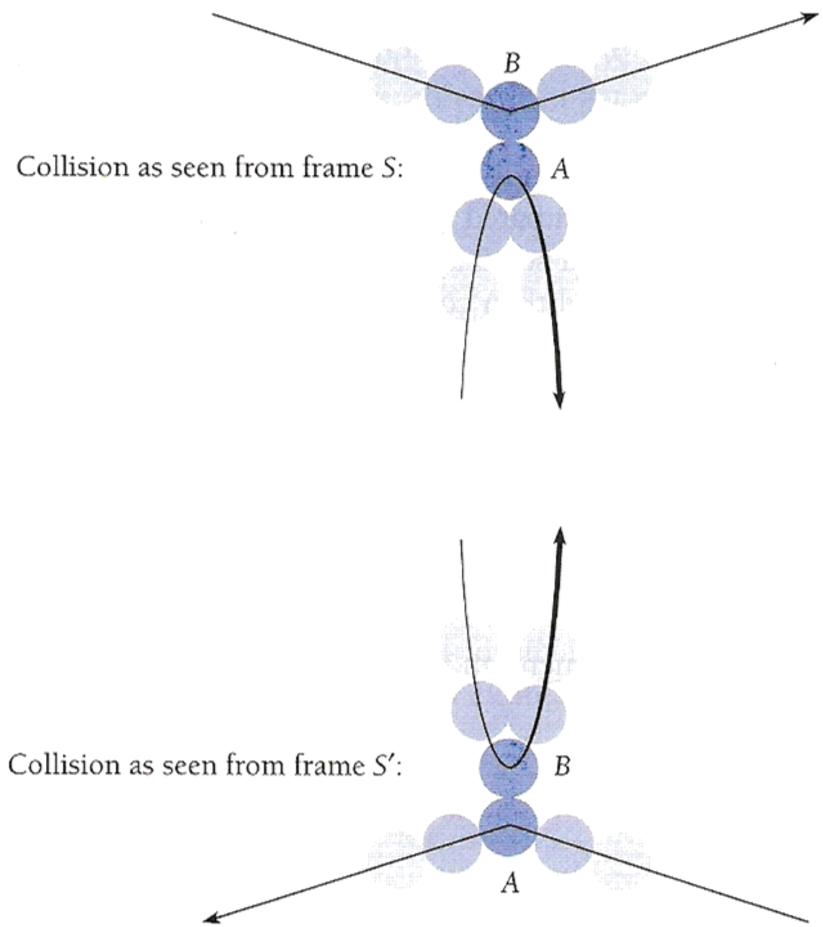
- We know that… $$T = \frac{T_o}{\sqrt{1-\upsilon^2/c^2}}$$



Observers in both frames see the same event but disagree about the length of time the particle thrown from the other frame requires to make the collision and return…

RELATIVISTIC MOMENTUM

How is the collision perceived in both frames…



RELATIVISTIC MOMENTUM

- **From S**
- the speed of B is: $V_B = \dfrac{Y}{T}$
- And $T = \dfrac{T_o}{\sqrt{1-\upsilon^2/c^2}}$
- Therefore, $V_B = \dfrac{Y\sqrt{1-\upsilon^2/c^2}}{T_o}$
- The speed of A is: $V_A = \dfrac{Y}{T_o}$

If we use the classical definition of momentum **p** = m**v**…

In frame S we have..

$$p_A = m_A V_A = m_A\left(\frac{Y}{T_o}\right)$$

$$p_B = m_B V_B = m_B\sqrt{1-\upsilon^2/c^2}\left(\frac{Y}{T_o}\right)$$

Will momentum be conserved in S if $m_A = m_B$?

RELATIVISTIC MOMENTUM

If we use the classical definition of momentum

p = m**v**...

In frame S we have..

$$p_A = m_A V_A = m_A \left(\frac{Y}{T_o} \right)$$

$$p_B = m_B V_B = m_B \sqrt{1 - \upsilon^2 / c^2} \left(\frac{Y}{T_o} \right)$$

Will momentum be conserved in S if $\mathbf{m_A = m_B}$?

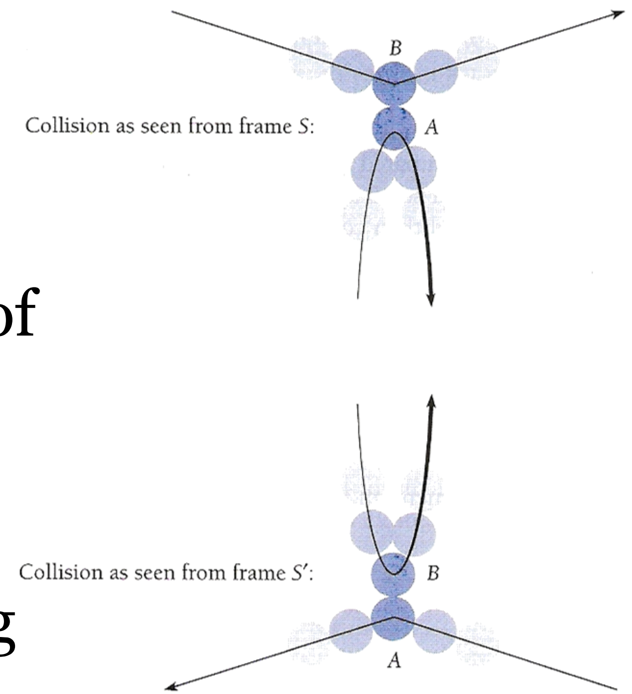
From S

momentum will be conserved ONLY if:

$$m_B = \frac{m_A}{\sqrt{1 - \upsilon^2 / c^2}}$$

RELATIVISTIC MOMENTUM

-If V_A and V_B' are very small compared to **v**, then…



From S

- B approach A with velocity **v**.
- In the limit V_A=0, m is the mass of A in S when A is at rest → $m_A = m$

- In the limit of V'_B=0, and m(v) is the mass of B in S, which is moving with velocity **v** → $m_B = m(v)$

$$m_B = \frac{m_A}{\sqrt{1-\upsilon^2/c^2}}$$

$$m(\upsilon) = \frac{m}{\sqrt{1-\upsilon^2/c^2}}$$

relativistic momentum

$$\mathrm{p} = \frac{m\upsilon}{\sqrt{1-\upsilon^2/c^2}}$$

RELATIVISTIC MOMENTUM

-When **v** << c then…

relativistic momentum

$$p = \frac{m\upsilon}{\sqrt{1-\upsilon^2/c^2}}$$

classical momentum

$$p = m\upsilon$$

m is the proper mass (or rest mass) ← when measured at rest relative to an observer.

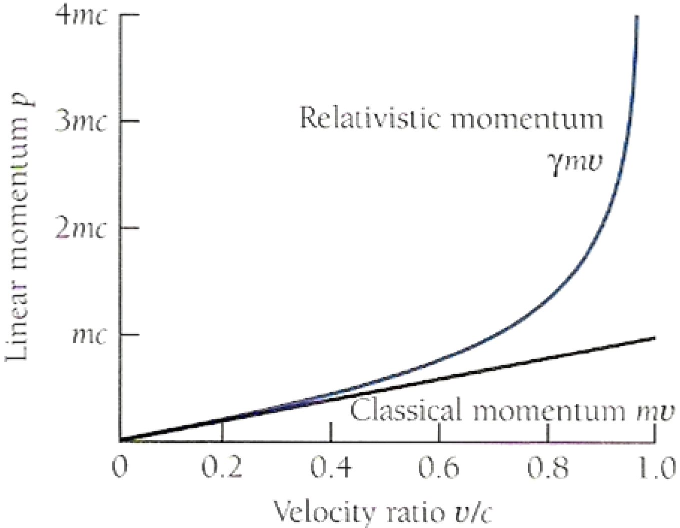
$$p = \gamma\, m\, v$$

$$\gamma = \frac{1}{\sqrt{1-\upsilon^2/c^2}}$$

RELATIVISTIC MOMENTUM

Will this affect Newton's second law?

$$\mathrm{F} = \frac{\mathrm{dp}}{\mathrm{dt}} = \frac{d}{dt}(\gamma\, m\, \mathrm{v})$$



RELATIVISTIC MOMENTUM

Remember…

Momentum is conserved and need to be redefined…

RELATIVISTIC MOMENTUM

Example 1.5:

Find the acceleration of a particle of mass *m* and velocity **v** when it is acted upon by the constant force **F**, where **F** is parallel to **v**.