Projectile Motion - Worksheet

From the given picture; you can see a skateboarder jumping off his board when he encounters a rod. He manages to land on his board after he passes over the rod.

1. What is the difference between the motion of the skateboard and the motion of the man (the skateboarder)?

2. How does the man manages to keep the skateboard underneath him, allowing him to land on it?

3. According to Newton's first law, the man should continue flying in the air. Why does he take a different path? What is this motion called?

4. Try to simulate the motion of the man using the device in front of you.

5. Can you suggest a name for this device?

6. What is the shape of the path taken by the fired ball?

7. What factors might affect the path of the ball?

Note: For all of the following experiments please use the short rang and use a plastic ball. For your safety please avoid looking at the muzzle and stay away from the path of the ball.

Part 1: free fall and projectile motion (ball fired horizontally)

1. Determine the time the ball would take when shot ***horizontally*** from a launcher until it reaches the floor and record its final position.

2. Determine the time the ball would take to free-fall the vertical distance from the launcher muzzle to the floor (starting from rest) and record its final position.

3. Compare the time and the vertical distance that you obtained from both step1 and step 2. Explain your answer.

4. Summarize the similarities and differences between the two kinds of motion. (Aid your answer with a sketch)

5. Determine the initial velocity of the ball (muzzle velocity) when shot horizontally from a launcher. Repeat 3 times and average your results.

Part 2:the range and the initial angle.

1. By firing a ball on the bench table (not to the ground), examine how changing the angle might affect the horizontal distance (in this case called the range) by collecting your data in a table and representing it graphically.

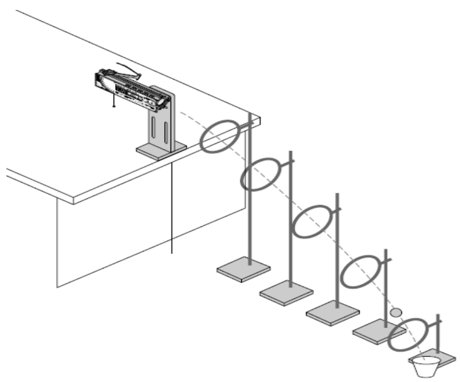
2. What angle gives the maximum range?

3. Compare the range when firing with an angle of 30° with the range when firing with an angle of 60°. What do you notice?

4. Use the data table of step 15 to determine the initial velocity. The range is related to the initial angle through the relation

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

Part 3:the path of the trajectory



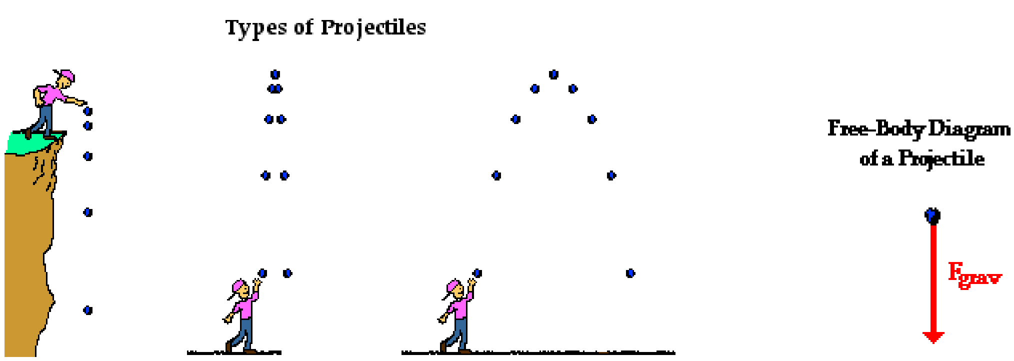
If you are given some rings. How can you use what you have learnt in this experiment to make the ball pass through all the rings successfully.
(use at least 3 rings-the cup is optional)

Projectile Motion

What is a Projectile?

Newton's laws of motion can be applied to objects moving in one or two dimension. The most common example of an object that is moving in *two dimensions* is a projectile.

A projectile is an object upon which the only force acting is gravity. There are a variety of examples of projectiles. An object dropped from rest is a projectile (provided that the influence of air resistance is negligible). An object that is thrown vertically upward is also a projectile (provided that the influence of air resistance is negligible). And an object which is thrown upward at an angle to the horizontal is also a projectile (provided that the influence of air resistance is negligible). A projectile is any object that once *projected* or dropped continues in motion by its own inertia and is influenced only by the downward force of gravity.

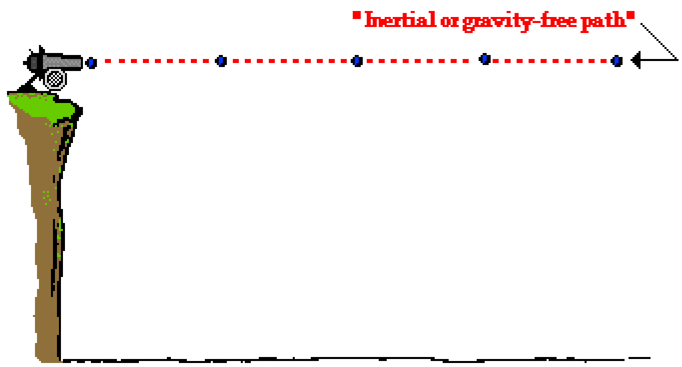


By definition, a projectile has a single force that acts upon it - the force of gravity. If there were any other force acting upon an object, then that object would not be a projectile. Thus, the free-body diagram of a projectile would show a single force acting downwards and labeled force of gravity (or simply F_{grav}). Regardless of whether a projectile is moving downwards, upwards, upwards and rightwards, or downwards and leftwards, the free-body diagram of the projectile is still as depicted in the diagram above. By definition, a projectile is any object upon which the only force is gravity.

Projectile Motion and Inertia

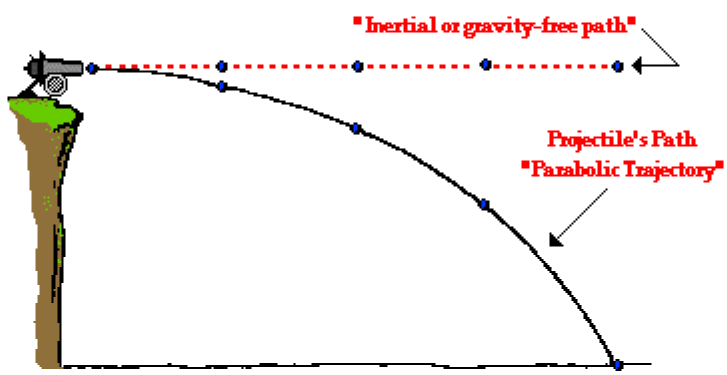
Many students have difficulty with the concept that the only force acting upon an upward moving projectile is gravity. Their conception of motion prompts them to think that if an object is moving upward, then there *must* be an upward force. And if an object is moving upward and rightward, there *must* be both an upward and rightward force. Their belief is that forces cause motion; and if there is an upward motion then there must be an upward force. They reason, "How in the world can an object be moving upward if the only force acting upon it is gravity?" Such students do not *believe* in Newtonian physics (or at least do not believe strongly in Newtonian physics). Newton's laws suggest that forces are only required to cause an acceleration (not a motion). Recall from the Unit 2 that Newton's laws stood in direct opposition to the common misconception that a force is required to keep an object in motion. This idea is simply not true! A force is not required to keep an object in motion. A force is only required to maintain an acceleration. And in the case of a projectile that is moving upward, there is a downward force and a downward acceleration. That is, the object is moving upward and slowing down.

To further ponder this concept of the downward force and a downward acceleration for a projectile, consider a cannonball shot horizontally from a very high cliff at a high speed. And suppose for a moment that the *gravity switch* could be *turned off* such that the cannonball would travel in the absence of gravity? What would the motion of such a cannonball be like? How could its motion be described? According to Newton's first law of motion, such a cannonball would continue in motion in a straight line at constant speed. If not acted upon by an unbalanced force, "an object in motion will ...". This is Newton's law of inertia.



In the absence of gravity, an object in motion will continue in motion with the same speed and in the same direction.

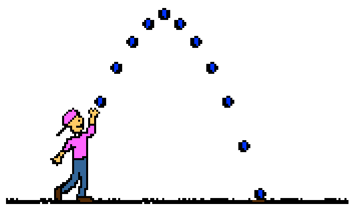
Now suppose that the *gravity switch* is turned on and that the cannonball is projected horizontally from the top of the same cliff. What effect will gravity have upon the motion of the cannonball? Will gravity affect the cannonball's horizontal motion? Will the cannonball travel a greater (or shorter) horizontal distance due to the influence of gravity? The answer to both of these questions is "No!" Gravity will act downwards upon the cannonball to affect its vertical motion. Gravity causes a vertical acceleration. The ball will drop vertically below its otherwise straight-line, inertial path. Gravity is the downward force upon a projectile that influences its vertical motion and causes the parabolic trajectory that is characteristic of projectiles.



With gravity, a "projectile" will fall below its inertial path. Gravity acts downward to cause a downward acceleration. There are no horizontal forces needed to maintain the horizontal motion - consistent with the concept of inertia.

A projectile is an object upon which the only force is gravity. Gravity acts to influence the vertical motion of the projectile, thus causing a vertical acceleration. The horizontal motion of the projectile is the result of the tendency of any object in motion to remain in motion at constant velocity. Due to the absence of horizontal forces, a projectile remains in motion with a constant horizontal velocity. Horizontal forces are not required to keep a projectile moving horizontally. The only force acting upon a projectile is gravity!

Characteristics of a Projectile's Trajectory

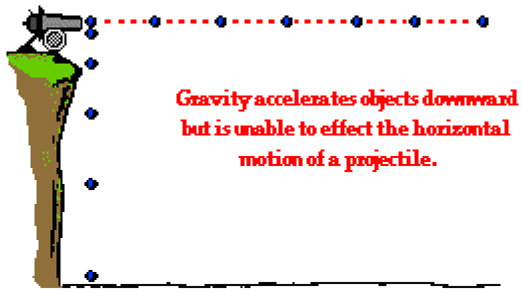


A projectile often moves horizontally as it moves upward and/or downward.

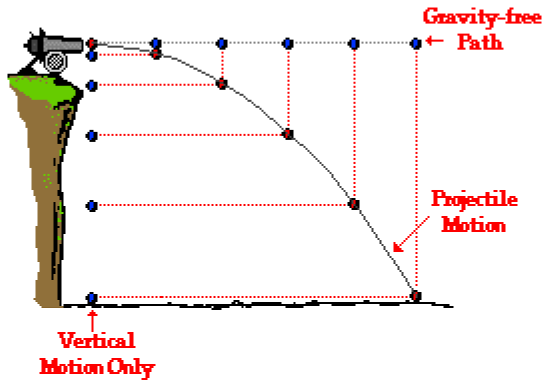
Many projectiles not only undergo a vertical motion, but also undergo a horizontal motion. That is, as they move upward or downward they are also moving horizontally. There are the two components of the projectile's motion - horizontal and vertical motion. And since perpendicular components of motion are independent of each other, these two components of motion can (and must) be discussed separately. The goal of this part is to discuss the horizontal and vertical components of a projectile's motion; specific attention will be given to the presence/absence of forces, accelerations, and velocity.

Horizontally Launched Projectiles

Let's return to our *thought experiment* mentioned above. Consider a cannonball projected horizontally by a cannon from the top of a very high cliff. In the absence of gravity, the cannonball would continue its horizontal motion at a constant velocity. This is consistent with the law of inertia. And furthermore, if merely dropped from rest in the presence of gravity, the cannonball would accelerate downward, gaining speed at a rate of 9.8 m/s every second. This is consistent with our conception of free-falling objects accelerating at a rate known as the **acceleration of gravity**.



If our thought experiment continues and we project the cannonball horizontally in the presence of gravity, then the cannonball would maintain the same horizontal motion as before - a constant horizontal velocity. Furthermore, the force of gravity will act upon the cannonball to cause the same vertical motion as before - a downward acceleration. The cannonball falls the same amount of distance as it did when it was merely dropped from rest (refer to diagram below). However, the presence of gravity does not affect the horizontal motion of the projectile. The force of gravity acts downward and is unable to alter the horizontal motion. There must be a horizontal force to cause a horizontal acceleration. (And we know that there is only a vertical force acting upon projectiles.) The vertical force acts perpendicular to the horizontal motion and will not affect it since perpendicular components of motion are independent of each other. Thus, the projectile travels with a constant horizontal velocity and a downward vertical acceleration.

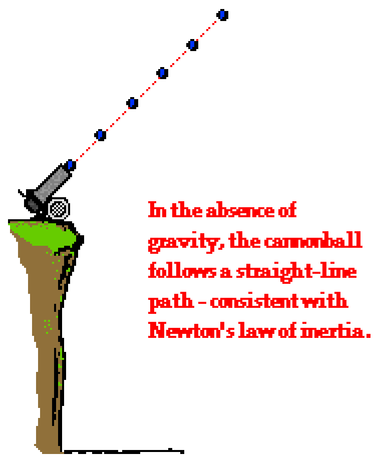


The above information can be summarized by the following table.

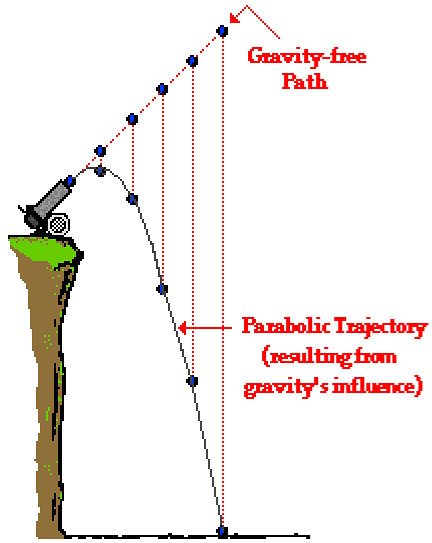
	Horizontal Motion	**Vertical Motion**
Forces (Present? - Yes or No) (If present, what direction?)	No	Yes The force of gravity acts downward
Acceleration (Present? - Yes or No) (If present, what direction?)	No	Yes "g" is downward at 9.8 m/s/s
Velocity (Constant or Changing?)	Constant	Changing (by 9.8 m/s each second)

Non-Horizontally Launched Projectiles

Now suppose that our cannon is aimed upward and shot at an angle to the horizontal from the same cliff. In the absence of gravity (i.e., supposing that the *gravity switch* could be *turned off*) the projectile would again travel along a straight-line, inertial path. An object in motion would continue in motion at a constant speed in the same direction if there is no unbalanced force. This is the case for an object moving through space in the absence of gravity. However, if the *gravity switch* could be *turned on* such that the cannonball is truly a projectile, then the object would once more *free-fall* below this straight-line, inertial path. In fact, the projectile would travel with a *parabolic trajectory*.



The downward force of gravity would act upon the cannonball to cause the same vertical motion as before - a downward acceleration. The cannonball falls the same amount of distance in every second as it did when it was merely dropped from rest (refer to diagram below). Once more, the presence of gravity does not affect the horizontal motion of the projectile. The projectile still moves the same horizontal distance in each second of travel as it did when the *gravity switch* was turned off. The force of gravity is a vertical force and does not affect horizontal motion; perpendicular components of motion are independent of each other.



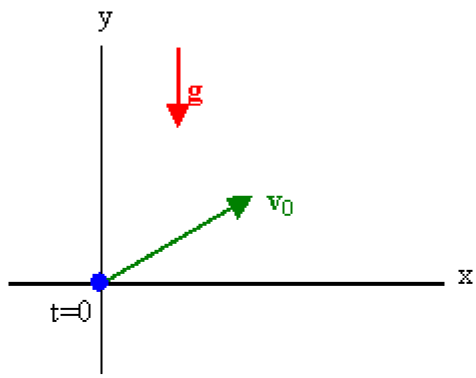
In conclusion, projectiles travel with a parabolic trajectory due to the fact that the downward force of gravity accelerates them downward from their otherwise straight-line, gravity-free trajectory. This downward force and acceleration results in a downward displacement from the position that the object would be if there were no gravity. The force of gravity does not affect the horizontal component of motion; a projectile maintains a constant horizontal velocity since there are no horizontal forces acting upon it.

Describing Projectile Motion Mathematically

Assume that we want to describe the motion of an object moving in projectile path, starting at time $t = 0$. Let us orient our coordinate system such that one of the axes, say the y-axis, points upward. Then:

$$\mathbf{a} = a_y\,\mathbf{j} = -\,g\,\mathbf{j}, \quad a_y = -g.$$

We can rotate our coordinate system about the y-axis until the velocity vector of the object at $t = 0$ lies in the x-y plane, and we can choose the origin of our coordinate system to be at the position of the object at $t = 0$.



Since we have motion with constant acceleration, the velocity at time t will be:

$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t,$

and the position will be:

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + (1/2)\mathbf{a}\, t^2.$

Writing this equation in component form and using,

$\mathbf{r}_0 = 0$, $\mathbf{v}_0 = v_{0x}\,\mathbf{i} + v_{0y}\,\mathbf{j}$, and $\mathbf{a} = a_y\,\mathbf{j} = -g\,\mathbf{j}$

we have:

$\mathbf{v} = v_{0x}\,\mathbf{i} + v_{0y}\,\mathbf{j} - g\,t\,\mathbf{j},$

$\mathbf{r} = v_{0x}\,t\,\mathbf{i} + v_{0y}\,t\,\mathbf{j} - (1/2)gt^2\,\mathbf{j}.$

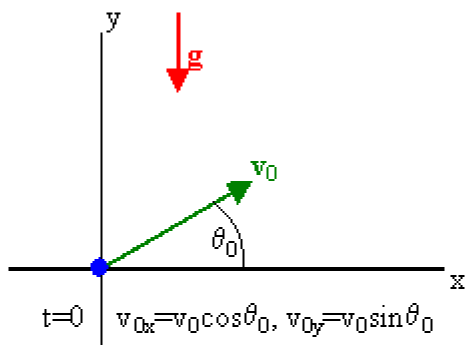
The position vector **r** and the velocity vector **v** at time t have only components along the *x*- and *y*-axes. By choosing a convenient orientation of our coordinate system we have simplified the mathematics involved in solving a projectile motion problem. We can treat projectile motion as **motion in two dimensions** with

$v_x = v_{0x}, \quad x = v_{0x}\,t,$

$v_y = v_{0y} - gt, \quad y = v_{0y}t - (1/2)gt^2.$

If the initial velocity $\mathbf{v}_0$ makes an angle θ_0 with the x-axis, then:

$v_{0x} = v_0 \cos\theta_0$ and $v_{0y} = v_0 \sin\theta_0$.



Then

$v_x = v_0\cos\theta_0 = constant,$ $x = v_0\cos\theta_0 t,$

$v_y = v_0\sin\theta_0 - gt,$ $y = v_0\sin\theta_0 t - (1/2)gt^2.$

We can solve $x = v_0 \cos\theta_0 t$ for t in terms of x, $t = x /(v_0 \cos\theta_0)$ and substitute this expression for t into $y = v_0 \sin\theta_0 t - (1/2) gt^2$. We obtain

$$y = x \tan(\theta_0) - gx^2/(2 v_0^2 \cos^2(\theta_0)),$$

an equation for the path (or **trajectory**) of the object. This equation is of the form $y = ax - bx^2$, which is the equation of a **parabola** which passes through the origin. The trajectory for projectile motion is a parabola.